Int. J. Solids Structures, Vol. 11, pp. 477–492. Pergamon Press 1975. Printed in Great Britain

FLUTTER OF HANGING ROOFS AND CURVED MEMBRANE ROOFS

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(*Received* 5 *July* 1974)

Abstract—This paper presents the determination method of flutter critical wind velocity of hanging roofs and curved membrane roofs of which outer surface only is exposed to wind, and some numerical results for fundamental shape of the roofs are illustrated to make clear the influences of curvatures, materials, size factors and prestressing on this critical velocity, and the dynamic behavior of the roofs subjected to wind. The procedure to formulate the governing equations is on the basis of classical flow theory with vorticities in the boundary layer and in the wake from the trailing edge, and the solution is obtained with Galerkin's procedure.

INTRODUCTION

Pneumatic membrane structures and hanging roofs are rather popular as architectures for the sake of easy construction and getting wide space economically, but they are of weak stiffness. Then the analysis of dynamic behavior is among the most important current research bearing on engineering reliability for them. This report is concerned with the determination of the critical wind velocity at which flutter occures in these roofs subjected to wind in the longitudinal direction. In the dynamic analysis of such structures as plates and shells, especially, as hanging roofs and pneumatic membrane structures subjected to wind, not only the influence from wind to structures but also the influence from structures to wind should be taken into account. That is, in flexible structures interaction between structure and stream should be considered.

There are many investigations with regard to only structural vibrations of such roofs subjected to predetermined external excitation forces available, for instance[1, 2], but are few investigations that have been made concerning this interaction problem. Otherwise, there are many contributions to panel and shell flutter available, for instance[3–5], for the calculation of aerodynamic forces due to supersonic flow[6], for bending-torsional fluttering of plate subjected to wind[7], for panel divergence at subsonic speeds, and elaborate reviewal work on this problem by Dowell[8]. But most investigations on panel and shell flutter are concerned with aeronautical structures and problems considered there are restricted in the range of supersonic or transonic, even in the case titled subsonic, air speeds with flow on both sides of the structures. These results, therefore, are not directly applicable to flutter problem of closed buildings subjected to low speed flow, i.e. at most several 10 m/sec. The experimental study carried out by Siev[9] for a model of closed building suggested the existance of the danger of flutter, where he observes flutter phenomena as vibrations of increasing amplitude induced at a certain air speed. However, this study did not give the quantitative indications for the designer of these structures.

If the boundary layer thickness of air is on the order of the flutter wave length, viscous effect will be important as stated by Dowell[8]. But since we deal with extremely low air speeds, low

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frequency flutter, that is, low order deflection mode vibration becomes significant also as shown in [8], and thence wave length is extremely large compared to the thickness of boundary-layer. Therefore wind can be assumed to be inviscid potential flow. But in order to determine the velocity field of ideal potential flow induced with the vibration of the roofs the treatment of the difference of tangential velocities between wind and boundary structures at boundary seems to become important, since Siev [9] intimated that flutter is associated with the appearance of vortices inducing pressure variation *vis-a-vis* the structures. Therefore to consider the existence of circulation which is equivalent to a discontinuity in the velocity seems rather realistic than to leave this difference as it is. Infinitesimally thin vortex sheet, thence, will be introduced between wind and roofs, and also in the wake from the trailing edge.

By introducing vortex sheets the aerodynamical damping effect on the vibrating roofs varies from positive to negative according to the increase of wind velocity for every deflection modes of the roofs, whereas this damping effect is positive or negative constantly according to the deflection modes without regard to wind velocity in ideal potential flow without vortex sheets. This indicates that by introducing vortex sheets the self-exciting vibration, that is, flutter of the roof can be determined at the wind velocity which is lower than divergence critical wind velocity. Above mentioned treatment appears just as the effect of boundary layer in real situation would be taken into consideration by replacing the boundary layer zone with vortex sheets. This assumption seems to be similar to classical wing theory. But in wing theory the wing itself is replaced by vorticities [10] because the wind is exposed both sides to air flow, while the roof considered here is exposed one side to wind. Therefore former results can not be directly applicable to this problem.

Wind and these vortex sheets will vary according to the movement of vibrating roofs, and simultaneously the roofs will be affected by this disturbed wind and these vortex sheets. Indoor air, if such structure as shown in sketch is considered, which is stationary in steady state will be forced to move in accordance with movement of the roofs, and this air flow also affects the roofs. In the present report the flutter critical wind velocity of curved membrane roofs and hinging roofs of no bending rigidity will be examined in scope of linear theory with aforementioned treatment of air flow.

Reference [8] shows a division of the Mach number, aspect ratio plane into various regions where different types of flutter predominate for the plates. Refering this, [3] and [9] appearance of flutter phenomena will be assumed when eigenvalue becomes complex number with negative damping effect with increased wind velocity in this report. Here the deflection is set as travering wave mode and with Galerkin's procedure an ordinary differential equation of vibration of the roof with time as the independent variable is derived. And then flutter critical wind velocity will be determined as the wind velocity at which eigenvalue takes real value, that is, positive damping vanishes with increased wind velocity. Since the roofs can be assumed to be stationary under the action of the steady wind of constant velocity U and vortex sheets due to U, it is sufficient to treat only the vibration in regard to fluctuating portions of disturbances from steady state. Divergence critical wind velocity will also be shown.

Although the author recognizes well that to take into consideration the effects of the turbulant structure of the wind and the separate flow from the front edge is essential in real situation, the inclusion of these effects complicate the problem awfully and to get the solution analytically may become impossible. Even the flow is idealized as aforementioned, the results given here will be of use as an indication for the engineers of these buildings. Moreover, since the length of the structure is usually very large, the results shown here would be adequate if separate flow from the front face is reattached in the sufficiently small distance from the front.

GOVERNING EQUATIONS

The roof is assumed to be shaped as

$$z = z(x, y), \qquad (0 \le x \le L, -y_0 \le y \le y_0)$$

under the action of pretensions N_{xo} and N_{yo} in static state ($t < 0$) and z being sufficiently small in comparison with reference length L (shallow assumption), with the following equilibrium condition.

$$N_{xo}\kappa_x + N_{yo}\kappa_y = 0 \qquad (t < 0) \tag{1}$$

where

$$\kappa_x = \frac{\partial^2 z}{\partial x^2}, \quad \kappa_y = \frac{\partial^2 z}{\partial y^2}, \quad \kappa_{xy} = \frac{\partial^2 z}{\partial x \partial y}.$$

That is,

$$t < 0: \quad u = v = w = 0, \quad N_{xy} = 0, \quad N_x = N_{xo}, \quad N_y = N_{yo}.$$

With the application of D'Alambert's principle and the assumption of shallow shape the kinematic equilibrium equation in state of flexural vibration of the roof of weight M per unit surface area, Young's modulus E and thickness h is

$$2N_{xy}(\kappa_{xy} + w_{,xy}) + N_x(\kappa_x + w_{,xx}) + N_y(\kappa_x + w_{,yy}) + P_z - \frac{M}{g}\ddot{w} = 0. \tag{2}$$

By setting stress resultants as

$$N_{xy} = \bar{N}_{xy}, \quad N_x = \bar{N}_x + N_{xo}, \quad N_y = \bar{N}_y + N_{yo}$$

and with the introduction of Airy-type stress function F as

$$\bar{N}_x = hF_{,yy}, \quad \bar{N}_y = hF_{,xx}, \quad \bar{N}_{xy} = -hF_{,xy}$$

and with the neglection of nonlinear terms equation (2) can be rewritten in the following form.

$$h(\kappa_x F_{,yy} + \kappa_y F_{,xx} - 2\kappa_{xy}F_{,xy}) + N_{xo}w_{,xx} + N_{yo}w_{,yy} + P_z - \frac{M}{g}\ddot{w} = 0. \tag{3}$$

Compatibility condition becomes

$$\nabla^2\nabla^2 F + E\{\kappa_x w_{,yy} - 2\kappa_{xy}w_{,xy} + \kappa_y w_{,xx}\} = 0. \tag{4}$$

DETERMINATION OF AERODYNAMIC FORCE

The variations of vorticities recognized over the roof which is exposed one side to wind only, unlike to the airfoil case, and in the wake flow from the trailing edge are not independent of the motion of the roof. Therefore the fluctuating normal load P_z due to wind should be determined with the consideration of the behavior of outside and inside air stream-roof interaction system. In this report wind is assumed as ideal potential flow.

(a) *Outside air flow*. The motion of the membrane roof wakes local small disturbance of velocity in the steady wind stream of constant velocity U in the vicinity of this roof. If the components of this local disturbance velocity are represented as $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$, and are assumed to be sufficiently small compared to U, the following interaction relation between disturbance velocity and the motion of the roof should be satisfied.

$$\tilde{w}_{+} \equiv \tilde{w}|_{z=0} = \dot{w} + (U + \tilde{u}|_{z=0})w_{,x} + \tilde{v}|_{z=0}w_{,y}$$

$$\simeq \dot{w} + Uw_{,x}. \tag{5}$$

Since velocity difference between wind and the roof exists, the appearance of turbulant boundary layer should be recognized in real situation. However, as the assumption of ideal potential flow is employed, this boundary layer zone will be replaced by the thin sheet of vortices of infinitesimal strength, dissimilarly to airfoil case where airfoil itself is replaced by the sheet of vortices. Also vortex sheet appears in the waked flow from the trailing edge. Otherwise, since only fluctuating portion from static state is of significance in the present calculation, only the treatment of the vortex sheets with respect to small distrubance velocity of wind becomes significant.

The distribution of bound vorticity per unit area (strength of circulation per unit area) over the roof and the distribution of free vortex sheet per unit longitudinal length per unit width in the waked flow, being represented as γ_c and γ_w, respectively, are expressed as the functions with respect to two spatial variables and variable time as follows,

$$\gamma_c = \gamma_c(x, y, t), \qquad \gamma_w = \gamma_w(x, y, t)$$

with the assumption that the sheets of variable vorticity are so thin that the thickness of these sheet can be negligible referred to airfoil case, and the deflection of the roof w is sufficiently small in comparison with other reference lengths of the structures.

It is well known that potential function can be defined in the stream layer containing vortices. Neglection of y-direction component of distrubance velocity leads the problem of determination of potential function in three dimensional flow system to that of two dimensional flow system in x–z plane, and therefore derivative of the potential function with respect to variable deduces the following relation for arbitrary y.

$$\tilde{u} - i\tilde{w} = \frac{1}{2\pi i}\int_{Z_0}^{Z_1} \frac{\gamma_c}{s - Z}\,\mathrm{d}Z + \frac{1}{2\pi i}\int_{Z_1}^{\infty} \frac{\gamma_w}{s - Z}\,\mathrm{d}Z \tag{6}$$

where

$$s = x + iz, \qquad i = \sqrt{(-1)}.$$

By setting $z = 0$,

$$\tilde{w}_{+} = \frac{1}{2\pi}\int_0^L \frac{\gamma_c}{x - S}\,\mathrm{d}S + \frac{1}{2\pi}\int_L^{\infty} \frac{\gamma_w}{x - S}\,\mathrm{d}S. \tag{7}$$

Equation (7) is of Hilbert integral equation and solution γ_c becomes as

$$\gamma_c = \frac{1}{\pi}\left(\frac{L-x}{x}\right)^{1/2}\left\{2\int_0^L \left[\frac{S}{L-S}\right]^{1/2}\frac{\tilde{w}_+}{S-x}\,\mathrm{d}S + \int_L^\infty \left[\frac{S}{S-L}\right]^{1/2}\frac{\gamma_w}{S-x}\,\mathrm{d}S\right\} + \frac{c}{\sqrt{[x(L-x)]}}. \tag{8}$$

From the condition that γ_c must be finite at the trailing edge ($x = L$), c has to vanish. Otherwise, from the definition of vorticity distribution γ_c can be given as the difference of velocity component $\tilde{u}_+$ and that of the roof. That is,

$$\gamma_c = \dot{u}_m - \tilde{u} \simeq \dot{u}_m - \tilde{u}_+ \tag{9}$$

where $\dot{u}_m$ is the velocity of the roof in x direction at that point and $\tilde{u}_+ = \tilde{u}|_{z=0}$.

Also velocity potential function can be defined in the small disturbance velocity field of the potential flow portion over the vortex sheet. Since it is assumed in the present treatment that vortex sheet on the roof is considerably thin, this function can be determined as follows.

$$\phi_+|_{z=0} = \int_{-\infty}^0 \tilde{u}_+\,\mathrm{d}x + \int_0^x \tilde{u}_+\,\mathrm{d}x. \tag{10}$$

(b) *Inside air flow*. Air inside the structure is assumed to have no velocity in the static state, and the movement of the roof wakes the indoor air flow. The inside of the structure is usually so large that the change of volume of inside air seems to be negligible. If velocity potential is defined for indoor air flow, it should satisfy the following equation.

$$\nabla^2\phi_- = 0. \tag{11}$$

According to the state of surrounding walls, boundary conditions of ϕ_- should be fixed, such as

$$\left.\frac{\partial\phi_-}{\partial z}\right|_{z=0} = \dot{w},\quad \left.\frac{\partial\phi_-}{\partial x}\right|_{x=L/2} = \left.\frac{\partial\phi_-}{\partial y}\right|_{y=\pm yo} = \left.\frac{\partial\phi_-}{\partial z}\right|_{z=-H} = 0. \tag{11$'$}$$

By applying Bernoulli's theorem, the variations of pressure of wind and inside air, which are of same density ρ_o, are

$$\Delta P_+ = -\rho_o\dot{\phi}_+ - \rho_o\tilde{u}U, \qquad \Delta P_- = -\rho_o\dot{\phi}_-$$

respectively, where terms which contain square of the components of small disturbance velocity are neglected. Thence, pressure difference P_z which acts as normal load in kinematic state is obtained as

$$P_z = -\rho_o(\phi_- - \phi_+)|_{z=0} + \rho_o\tilde{u}_+U. \tag{12}$$

SOLUTION

Since number of unknowns exceeds one to that of equations in aforementioned relations, an additional condition is required. Then the following assumption will be employed here.

As the assumption of ideal potential flow is used here, the variation of the circulation of vortices over the roof with respect to time is sent forth with the velocity equal to wind as the distribution of the free vorticity without decrease. That is,

$$\dot{\Gamma}_c \, \mathrm{d}t + \gamma_w(L, y, t) U \, \mathrm{d}t = 0 \quad \text{where} \quad \Gamma_c = \int_0^L \gamma_c \, \mathrm{d}x. \tag{13}$$

Otherwise, the sum of circulations with regards to γ_c and γ_w should vanish.

$$\int_0^L \gamma_c \, \mathrm{d}x + \int_L^\infty \gamma_w \, \mathrm{d}x = 0. \tag{14}$$

The variation of this equation with respect to time yields

$$\dot{\Gamma}_c \, \mathrm{d}t + \frac{\mathrm{d}}{\mathrm{d}t}\left\{\int_L^\infty \gamma_w \, \mathrm{d}x\right\} \mathrm{d}t = 0. \tag{15}$$

From the comparison of equation (13) with equation (15) the following condition can be deduced.

$$\dot{\gamma}_w(x, y, t) = -U \frac{\partial}{\partial x} \gamma_w(x, y, t)$$

or

$$\gamma_w(L, y, t) = \gamma_w\left(x, y, t + \frac{x - L}{U}\right) \tag{16}$$

with the assumption that

$$\gamma_w(\infty, y, t) = 0. \tag{17}$$

Equation (16), which is deduced on the basis of assumption equation (17), means that the change of the distribution of free vorticity in the wake flow is harmonic, and the employment of equation (16) as one more condition simplify the solution extremely. Despite the adequency of the assumption equation (17) can not be shown, usage of equation (16) seems to be recognisable because equation (16) is rigorous condition in state of harmonic vibration of the roof.

Now ten unknowns can be determined as the solutions of ten simultaneous differential equation identically. To get the analytic solution, however, is extremely difficult and therefore approximation procedure should be introduced, practically. Application of Galerkin's procedure seems more suitable than direct numerical calculation for the investigation given here.

In this report traveling wave type deflection mode of the following form, which includes wave propagation velocity as parameter, will be considered for simply supported boundary.

$$w = ALi\{EX(i\bar{\alpha}_1) - EX(j\bar{\alpha}_2)\} \, \mathrm{e}^{i\omega t} \cos \bar{\beta} y \tag{18}$$

where

$$EX(\alpha) = \mathrm{e}^{-\alpha x}$$

and

$$\bar{\alpha}_1 = \frac{\lambda}{kL} + \frac{m\pi}{L}, \quad \bar{\alpha}_2 = |\bar{\bar{\alpha}}_2|; \quad \bar{\bar{\alpha}}_2 = \frac{\lambda}{kL} - \frac{m\pi}{L}, \quad \bar{\beta} = \frac{n\pi}{2y_0}$$

$$m,\ n,\ \text{integer},\qquad \lambda = \frac{\omega L}{2U},\qquad j = \begin{cases} i \ldots \bar{\alpha}_2 \geq 0 \\ -i \ldots \bar{\alpha}_2 < 0 \end{cases}$$

k, nondimentional wave propagation velocity parameter, that is, wave propagation velocity is given as $u_0 = 2kU$.

By substitution of equation (18) into equation (4), stress function can be obtained as follows,

$$F = AELi\{K_1EX(i\bar{\alpha}_1) - K_2EX(j\bar{\alpha}_2)\}\, e^{i\omega t} \cos \bar{\beta} y \tag{19}$$

where

$$K_1 = \frac{L^2(\kappa_y\alpha_1{}^2 + \kappa_x\xi^2\beta^2)}{(\alpha_1{}^2 + \xi^2\beta^2)^2},\quad K_2 = \frac{L^2(\kappa_y\alpha_2{}^2 + \kappa_x\xi^2\beta^2)}{(\alpha_2{}^2 + \xi^2\beta^2)^2}\quad \text{and}\quad \xi = \frac{L}{2y_o},$$

$$\alpha_l = \bar{\alpha}_l L \qquad \beta = \bar{\beta}\,.\,2y_o.$$

In order to satisfy other ordinary boundary conditions it is sufficient to give the deflection w as the sum of expression (18) of suitable number with different m and n.

Whereas potential function of indoor air should have complicated form so as to satisfy boundary conditions rigorously, the following approximation will simplify the determination of potential function. Since it can be seen from equations (11) and (11)′ that the motion of indoor air becomes maximum at $z = 0$ and diminishes rapidly with getting away from the roof, the remainder of the boundary conditions (11)′ will be satisfied only at $z = 0$.

That is,

$$\left.\frac{\partial\phi_-}{\partial x}\right|_{\substack{x=L/2\\ z=0}} = \left.\frac{\partial\phi_-}{\partial y}\right|_{\substack{y=\pm y_o\\ z=0}} = 0. \tag{11)''$$

Then potential function of indoor air ϕ_- should have the following form in order to satisfy equation (11) and boundary condition (11)″.

$$\phi\ = -ALH\omega\{EX(i\bar{\alpha}_1)Z_1(z) - EX(j\bar{\alpha}_2)Z_2(z)\}\, e^{i\omega t} \cos \bar{\beta} y \tag{20}$$

and function $Z_l(z)$ can be determined as

$$Z_l(z) = C_l \cosh \eta_l(z/H + 1) \tag{21}$$

where

$$\eta_l = \frac{H}{L}\sqrt{(\alpha_l^2 + \xi^2\beta^2)},\qquad C_l = 1/(\eta_l \sinh \eta_l),\qquad l = 1, 2.$$

Substitution of equation (8) into equation (14) derives finally,

$$2\int_0^L \left[\frac{S}{L-S}\right]^{1/2} \bar{w}_+\, dS + \int_L^\infty \left[\frac{S}{S-L}\right]^{1/2} \gamma_w\, dS = 0. \tag{22}$$

According to the assumption equation (16) γ_w seems to have the following form,

$$\gamma_w = \bar{A}U\, e^{-i2\lambda(x-L)/L}\, e^{i\omega t} \cos \bar{\beta} y. \tag{23}$$

Refering to equations (5), (18) and (22), $\bar{A}$ can be given as

$$\bar{A} = \frac{4}{F(\lambda)} A\{(\alpha_1 - 2\lambda)[J_o(\alpha_1/2) - iJ_1(\alpha_1/2)]\, e^{-i(\lambda+\alpha_1/2)} + (i\,.\,j\,.\,\alpha_2 + 2\lambda) \times [J_o(\alpha_2/2) - jJ_1(\alpha_2/2)]\, e^{-(i\lambda + j.\alpha_2/2)}\}. \quad (24$$

Where

$$F(\lambda) = Y_o(\lambda) + J_1(\lambda) - i\{Y_0(\lambda) - J_0(\lambda)\}.$$

Equations (8), (5) and (23) determine γ_c.

Since $\dot{u}_m$ seems to be negligible compared to $\bar{u}_+$ in equation (9) and $\bar{u}_+(x<0)$ can be set to be zero in this problem, potential function of wind stream can be determined as

$$\dot{\phi}_+|_{z=0} = \frac{d}{dt}\int_o^x \bar{u}_+\, dx = -\int_o^x \dot{\gamma}_c\, dx. \quad (25)$$

Whence aerodynamical normal load P_z rewritten as equation (26) becomes determinate except ω.

$$P_z = -\rho_o\dot{\phi}_-|_{z=0} - \rho_o U\gamma_c - \rho_o\int_o^x \dot{\gamma}_c\, dx. \quad (26)$$

Application of Galerkin's procedure for equation (3) yields an algebraic equation associated with ω in the following form.

$$-Q_o\lambda^2 + \frac{1}{4V^2}Q_1 + \eta(-Q_2\lambda^2 + Q_3\lambda + Q_4) = 0 \quad (27)$$

where

$$V = U\sqrt{\frac{M}{Ehg}} = U\sqrt{\frac{\rho_m}{E}}, \quad \eta = \frac{\rho_o Lg}{M} = \frac{\rho_o L}{\rho_m h}, \quad \rho_m\text{: equivalent material density}$$

$$\lambda = \frac{\omega L}{2U} = \frac{\bar{\omega}}{V} \text{ and thence } \bar{\omega} = \frac{\omega L}{2}\sqrt{\frac{M}{Ehg}} = \frac{\omega L}{2}\sqrt{\frac{\rho_m}{E}}$$

and $Q_i = Q_i(\alpha_1, \alpha_2)$ are functions of ω. Some integral results are shown in Appendix.

Whence, such wind velocity that gives real positive roots of equation (27) is flutter critical wind velocity, and for that sake real positive λ should satisfy the following equation

$$I_m\{(-Q_2\lambda^2 + Q_3\lambda + Q_4)\,.\,\bar{Q}_1\} = 0 \quad (28)$$

because the multiple $Q_o\,.\,\bar{Q}_1$ takes real value, where $\bar{Q}_p$ represents complex conjugate of Q_p. Note that real positive root λ of equation (28), which can be considered as flutter critical parameter, is independent of the size and material factors of the structure. And also note that when ω takes real value the following relations $Q_p(-\omega) = Q_p(\omega)$ $(p \neq 3)$, $Q_3(-\omega) = -Q_3(\omega)$ can be derived and this indicates the adequency of the employment of real positive rooot of equation

(28). Thence, flutter critical wind velocity can be calculated as

$$4V_{cr}^2 = \frac{Q_1 \cdot \bar{Q}_1}{R_e[Q_o \cdot \bar{Q}_1\lambda^2 + \eta(Q_2\lambda^2 - Q_3\lambda - Q_4) \cdot \bar{Q}_1]} \tag{29}$$

in which λ is the real positive root of equation (28).

Required solution of equation (28) will be calculated as follows. Decreasing λ from extremely large real value, real positive solution $\lambda = \lambda_o$ can be found by trial and error method. Check whether λ_o provides real V_{cr} from equation (29). Then, by setting as $\lambda = \lambda_o - \Delta\lambda_o - i\bar{\lambda}$ and by calculating $Q_l \equiv Q_l(\lambda_o - \Delta\lambda_o)$, $\bar{\lambda}$ will be determined from equation (27). If $\bar{\lambda}$ is positive, λ_o is solution required.

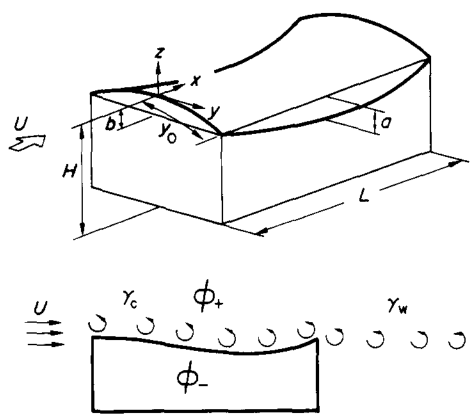
Statical divergence phenomena appear when $V \neq 0$, $k \neq 0$ and $\lambda \to 0$. In this state $\gamma_w = \dot{\gamma}_c = 0$ and $\gamma_c \neq 0$, and than $\bar{A} = 0$. Then Q_1 and Q_4 take real values. From equation (27) divergence critical wind velocity can be calculated as follows.

$$4V^2 = -\frac{Q_1}{\eta Q_4}. \tag{30}$$

It is evident that this critical velocity is higher than the determined from equation (29).

NUMERICAL EXAMPLE

As a example the most fundamental and popular shape as hanging roof given in the next form will be considered.



Sketch of structures and coordinate system.

$$z = \frac{4a}{L^2}(x^2 - Lx) - \frac{b}{y_o^2}y^2. \tag{31}$$

Then K_1 and K_2 included in Q_1 are given as follows.

$$K_l \equiv \bar{K}_l(\kappa_y\alpha^2 + \kappa_x\xi^2\beta^2)/L = 64\frac{(\bar{\kappa}_x\xi^2\beta^2 - \bar{\kappa}_y\xi\alpha_l^2)^2}{(\alpha_l^2 + \xi^2\beta^2)^2}, \quad l = 1, 2$$

and

$$\bar{\kappa}_x = a/L, \qquad \bar{\kappa}_y = b/2y_o.$$

It is well recognized that relatively lower $\bar{\omega}$ and flutter critical wind velocity V_{cr} are more significant in the design of these hanging roofs, and Fig. 1, which presents ($\bar{\omega} \sim V_{cr}$) relations associated with some different m and η in the case that $\bar{\kappa}_x = 0{\cdot}05$, $\bar{\kappa}_y = 0{\cdot}05$ and $\xi = 2$, indicates

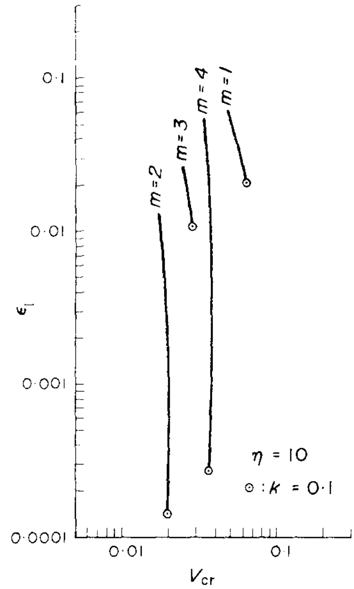


Fig. 1. Dependence of flutter critical wind velocity and frequency on deflection modes in the case ($\bar{\kappa}_x = \bar{\kappa}_y = 0{\cdot}05$, $\xi = 2$).

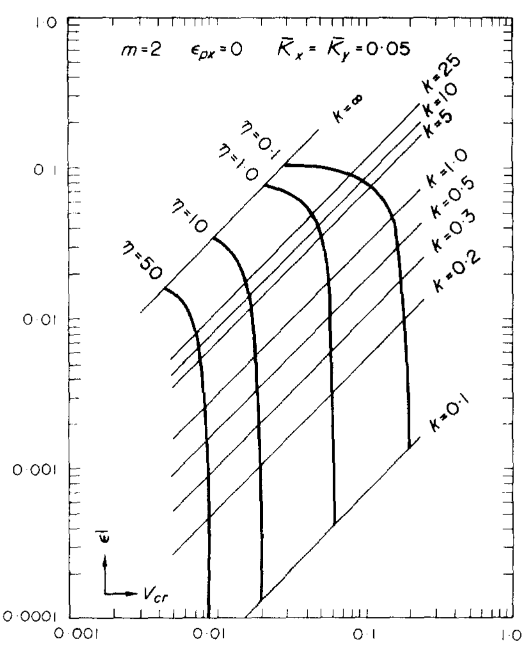


Fig. 2. Dependence of flutter critical wind velocity and frequency on traveling wave propagation speed.

that the case $m = 2$ is the most significant, despite the appearance of the minimum of V_{cr} depends on the deflection modes and $\bar{\omega}$ generally and is not necessarily in the case $m = 2$. Moreover, it was well ascertained that this tendency does not depend on the prestressing, if exists, and scarcely ever depends on the curvatures. Therefore, the case $m = 2$ will be illustrated in the following.

The variation of ($\bar{\omega} \sim V_{cr}$) curves with respect to the variation of k and η are shown in Fig. 2, and also ($V_{cr} \sim \eta$) relations associated with k are given without prestressing in Fig. 3. Whereas $\bar{\omega}$

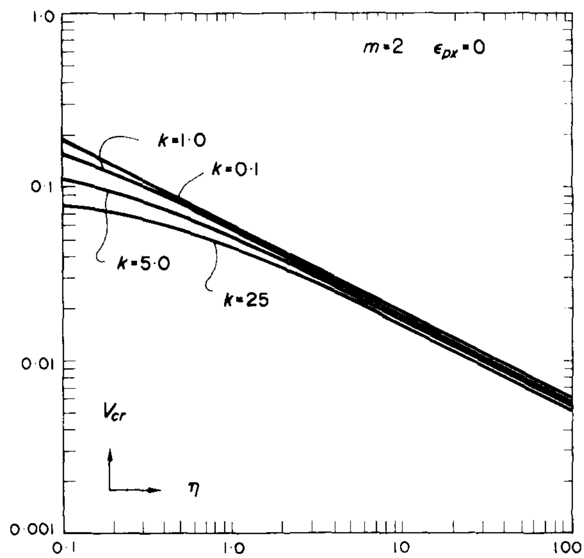


Fig. 3. Variation of flutter critical wind velocity due to weight of the roof and length.

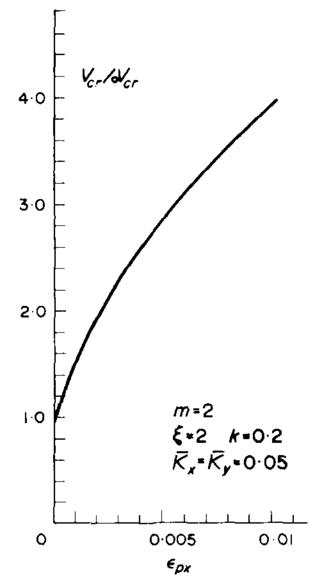


Fig. 4. Influence of prestress on flutter critical wind velocity, where ${}_0V_{cr}$ is that with no prestress.

increases and V_{cr} decreases with increased k for any η, the variation of V_{cr} becomes slight gradually against the variation of k according to the increase of η. When modified wind velocity V varies in the range where ($\bar{\omega} \sim V_{cr}$) curves is given for a η, a harmonic neutral vibration of the roof can be determinate certainly, and then it seems the true flutter critical wind velocity to be defined as the V_{cr} which is determined at the limit $\lambda/k \to 0$ with $k \to 0$, and where no harmonic neutral vibration exists. That is, before the fluttering appears the roof keeps repose, and when wind velocity V gets to the true V_{cr} the roof changes from the stationary state into dynamic state. The limit $\lambda/k \to 0$ with $k \to 0$ gives singular point of the aforementioned governing equation (27) and V_{cr} can not be determined, but Fig. 2 indicates that in the range of considerably small k V_{cr} is asymptotical to a constant value and v_{cr} correspond to $k = 0{\cdot}2$ becomes good approximate to the true flutter critical wind velocity.

V_{cr} increases with the existance of prestressing. The influence of prestressing on V_{cr} is shown in Fig. 4 for a case of curvature set. In Fig. 4 the magnitude of the prestressing is represented as corresponding strain as $\epsilon_{px} = N_{xo}/Eh$ and $\epsilon_{py} = N_{yo}/Eh$. When ϵ_{px} is predetermined ϵ_{py} can be calculated from equation (1), and vice versa. Note that this curve is independent of η and dependent on curvatues and λ/k. But, as the maximum deviation from this curve due to the change of k is within $\pm 0{\cdot}05\%$ in the range from $k = 0{\cdot}2$ to $k \to \infty$, this curve can be applicable for any k. Flutter critical parameter λ is dependent on k and m as shown in Fig. 5, and $\bar{\omega}$ can be determined from this curve and V_{cr}. In the limiting case $k \to \infty$ (steady deflection mode), λ takes as 3·6726.

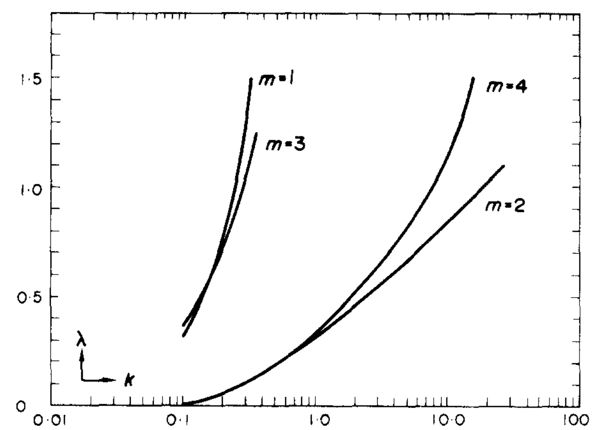


Fig. 5. Dependence of flutter critical parameter λ on traveling wave propagation speed.

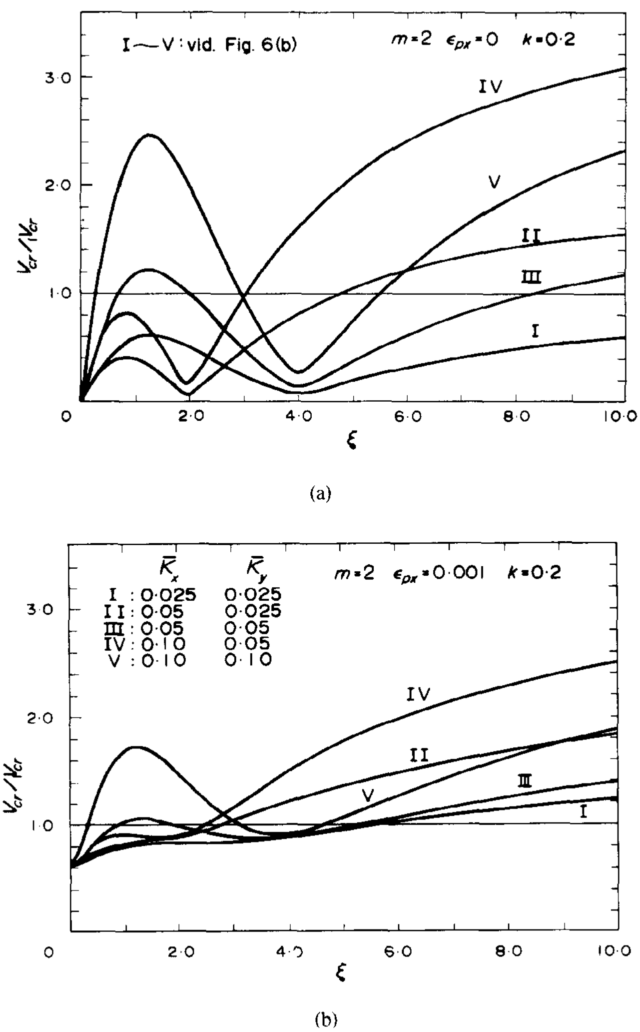
In Fig. 6 V_{cr} with different sets of curvatues and length-width ratio ξ are illustrated under the standerdization with V_{cr} of the case $\bar{\kappa}_x = 0{\cdot}05$, $\bar{\kappa}_y = 0{\cdot}05$ and $\xi = 2$. Therefore, flutter critical state of hanging roofs of any material, any prestressing, any scales and size and any curvatures in the range shown in Fig. 6 can be clarified from these figures.

The two dimensional theory treated here is appropriate for large width/length ratio structures and they correspond to small ξ in Figs. 6.

An example with practical values

Roof material: *Nylon cloth membrane.* $E = 3 \times 10^7$ kg/m^2, $\rho_m = 117{\cdot}35$ kg . sec^2/m^4, $a/L = 0{\cdot}05$, $b/2y_o = 0{\cdot}05$, $\xi = 2$, $H/L = 0{\cdot}3$, $\rho_o = 0{\cdot}12296$ kg . sec^2/m^4, $m = 2$, $k = 0{\cdot}2$

h/L	N_{xo}	N_{yo}	ωL (m/sec)	U_{cr} (m/sec)
0·0001	0	0	1·0375	9·38
	0·001	0·0005	1·6334	14·78
0·001	0	0	3·2794	29·67
	0·001	0·0005	5·1628	46·70



Figs. 6. Influence of curvature and length-width ratio on flutter critical wind velocity, ${}_1V_{cr}$ in (a) and (b): flutter critical wind velocity in the case ($\bar{\kappa}_x = \bar{\kappa}_y = 0{\cdot}05$, $\xi = 2$) with no prestress and $\epsilon_{px} = 0{\cdot}01$, respectively.

Acknowledgements—The author wishes to express his gratitude to the chairman, Emer. Prof. Y. Tsuboi, and other members of the subcommittee of Suspension Structures in the Society of Steel Construction of Japan for their frequent, stimulating and helpful discussions on this work.

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APPENDIX

Definite integral representation of Q_i which constitutes equation (27) is in need of the numerical calculation, because some Q_i is defined by infinite integrals having singular point and direct numerical integration procedure is not applicable. Q_i can be calculated from the following functions.

$$Q_o = \sum_{p=1}^{2} \sum_{q=1}^{2} (-1)^{p+q} R(p, q)$$

$$Q_1 = \sum_{p=1}^{2} \sum_{q=1}^{2} (-1)^{p+q} (K_p + \epsilon_{px}\alpha_p{}^2 + \epsilon_{py}\xi^2\beta^2) R(p, q) \qquad \epsilon_{px} = N_{xo}/Eh, \quad \epsilon_{py} = N_{yo}/Eh$$

$$Q_2 = \sum_{p=1}^{2} \sum_{q=1}^{2} (-1)^{p+q} \left\{ \frac{H}{L} Z_p(o) R(p, q) + \frac{2}{\pi} S(p, q) \right\}$$

$$Q_3 = \frac{1}{\pi} \sum_{p=1}^{2} \sum_{q=1}^{2} (-1)^{p+q} \left\{ iW(p, q) + \left(\frac{j}{i}\right)^{p-1} \alpha_p S(p, q) \right\} + \frac{1}{2\pi} \bar{A}\, e^{i2\lambda} \{Y(1) - Y(2)\}$$

$$Q_4 = -\frac{i}{2\pi} \sum_{p=1}^{2} \sum_{q=1}^{2} (-1)^{p+q} \left(\frac{j}{i}\right)^{p-1} \alpha_p W(p, q) - \frac{i}{4\pi} \bar{A}\, e^{i2\lambda} \{Y(1) - Y(2)\}$$

where

$$R(p, q) = \frac{1}{\alpha_p i_p + \alpha_q i_q} (1 - e^{-i(2\lambda/k)})$$

if

$$\lambda/k \to 0; \quad R(p, q) = 1 \quad (p \neq q)$$
$$= 0 \quad (p = q)$$

$$S(p, q) = \frac{\pi^2}{2\alpha_q} e^{-(i_p\alpha_p + i_q\alpha_q)/2} \left[J_o\left(\frac{\alpha_p}{2}\right) \left\{ J_1\left(\frac{\alpha_q}{2}\right) - i_q J_o\left(\frac{\alpha_q}{2}\right) \right\} + i_q\, e^{-i_q\alpha_q/2} \left\{ J_o\left(\frac{\alpha_p}{2}\right) - i_p J_1\left(\frac{\alpha_p}{2}\right) \right\} \right.$$
$$\left. + \frac{4}{\alpha_q} \sum_{r=1}^{\infty} (i_p i_q)^{-r} r J_r\left(\frac{\alpha_p}{2}\right) J_r\left(\frac{\alpha_q}{2}\right) \right] \quad \text{if} \quad \alpha_q = 0;$$

$$S(p,q)=\pi^2 e^{-i_p(\alpha_p/2)}\left\{3J_o\left(\frac{\alpha_p}{2}\right)-i_p\,.\,2J_1\left(\frac{\alpha_p}{2}\right)+J_2\left(\frac{\alpha_p}{2}\right)\right\}\Big/8$$

$$W(p,q)=\pi^2 e^{-(i_p\alpha_p+i_q\alpha_q)/2}\left[\frac{1}{2}J_o\left(\frac{\alpha_p}{2}\right)\left\{J_o\left(\frac{\alpha_q}{2}\right)+i_qJ_1\left(\frac{\alpha_q}{2}\right)\right\}+\frac{2}{\alpha_q}\sum_{r=1}^{\infty}i_p^{-r}i_q^{1-r}rJ_r\left(\frac{\alpha_p}{2}\right)J_r\left(\frac{\alpha_q}{2}\right)\right]$$

if $\alpha_q=0$;

$$W(p,q)=\frac{\pi^2}{2}e^{-i_p\alpha_p/2}\left\{J_o\left(\frac{\alpha_p}{2}\right)-i_pJ_1\left(\frac{\alpha_p}{2}\right)\right\}.$$

A. Erdélyi has given the following integral[11],

$$F(\lambda,\theta)\equiv\frac{1}{2\pi}\int_o^{\infty}e^{i\lambda\cosh t}\frac{\sin\theta}{\cosh t+\cos\theta}\,dt$$
$$=\frac{\theta}{2\pi}e^{-i\lambda\cos\theta}+\frac{1}{2}\sum_{n=1}^{\infty}i^{-n-1}\sin\theta\left\{J_n(\lambda)-\frac{2}{\pi i}\left[\frac{\partial J_\nu(\lambda)}{\partial\nu}\right]_{\nu=n}\right\}.$$

Applying this result to the two integrals of the following type,

$$\int_o^1\left(\frac{1-x}{x}\right)^{1/2}e^{-i\alpha x}\int_1^{\infty}\left(\frac{\eta}{\eta-1}\right)^{1/2}\frac{1}{\eta-x}e^{-i\beta\eta}\,dx\,d\eta$$

and

$$\int_o^1 e^{-i\alpha x}\int_o^x\left(\frac{1-\eta}{\eta}\right)^{1/2}\int_1^{\infty}\left(\frac{\xi}{\xi-1}\right)^{1/2}\frac{1}{\xi-\eta}e^{-i\beta\xi}\,dx\,d\eta\,d\xi$$

functions $X(p)$ and $Y(p)$ can be represented as follows.

$$X(p)=-\pi^2 e^{-(i\lambda+i_p\alpha_p/2)}\left[\frac{1}{4}\left\{J_o\left(\frac{\alpha_p}{2}\right)+i_pJ_1\left(\frac{\alpha_p}{2}\right)\right\}\{Y_o(\lambda)+iJ_o(\lambda)\}\right.$$
$$\left.-\frac{1}{2\pi(i\lambda+i_p\alpha_p/2)}\left\{J_o\left(\left|\lambda+\frac{i_p}{i}\frac{\alpha_p}{2}\right|\right)-e^{-(i\lambda+i_p\alpha_p/2)}\right\}+\frac{1}{\alpha_p}\sum_{r=1}^{\infty}(ii_p)^{r-1}rJ_r\left(\frac{\alpha_p}{2}\right)\cdot f_r(\lambda)\right]$$

if $i\lambda+i_p\alpha_p/2=0$; the 2nd term in brackets[ ] to be replaced by $-1/2\pi$. If $\alpha_p=0$;

$$X(p)=-\pi^2 e^{-i\lambda}\left[\{Y_o(\lambda)+iJ_o(\lambda)\}/4+\frac{i}{2\pi\lambda}\{J_o(\lambda)-e^{-i\lambda}\}+\frac{1}{4}f_1(\lambda)\right]$$

$$Y(p)=\frac{\pi}{4}e^{-(i\lambda+i_p\alpha_p/2)}\left[-\frac{\pi}{\alpha_p}\left\{J_1\left(\frac{\alpha_p}{2}\right)-i_pJ_o\left(\frac{\alpha_p}{2}\right)+i_p\,e^{-i_p\alpha_p/2}\right\}\right.$$
$$+\frac{1}{\lambda(i\lambda+i_p\alpha_p/2)}\left\{J_o\left(\left|\lambda+\frac{i_p}{i}\frac{\alpha_p}{2}\right|\right)-e^{-(i\lambda+i_p(\alpha_p/2))}\right\}$$
$$+i_p\frac{2}{\alpha_p}\left\{\frac{i}{\lambda}J_o(\lambda)+\frac{\pi}{2}f_1(\lambda)\right\}\left\{J_o\left(\frac{\alpha_p}{2}\right)-e^{-i_p\alpha_p/2}\right\}$$
$$\left.+\frac{4}{\lambda\alpha_p}\sum_{r=1}^{\infty}(ii_p)^{r-1}J_r\left(\frac{\alpha_p}{2}\right)J_r(\lambda)+i_p\frac{4\pi}{\alpha_p^2}\sum_{r=1}^{\infty}(ii_p)^{r-1}rJ_r\left(\frac{\alpha_p}{2}\right)f_r(\lambda)\right]$$

if $i\lambda + i_p\alpha_p/2 = 0$; the 2nd term in brackets [ ] to be replaced by $1/2\pi\lambda$. If $\alpha_p = 0$;

$$Y(p) = \frac{\pi}{4} e^{-i\lambda} \left[\frac{3}{4} \pi \{ Y_o(\lambda) + iJ_o(\lambda) \} + \frac{1}{\lambda^2} \{ J_o(\lambda) - e^{-i\lambda} \} \right.$$
$$\left. - \frac{i}{\lambda} J_o(\lambda) + \frac{1}{\lambda} J_1(\lambda) - \frac{\pi}{2} f_1(\lambda) - i \frac{\pi}{4} f_2(\lambda) \right]$$

and where

$$i_1 = i, \qquad i_2 = j$$

$$f_s(\lambda) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(n+s)!} \left(\frac{\lambda}{2} \right)^{s+2n} \left\{ 1 + \frac{2}{i\pi} \log \frac{\lambda}{2} - \frac{2}{i\pi} \psi(s+n+1) \right\}$$

$\psi(m)$, Gaussian ψ-function.